

How Wealthy Are Europeans?

Grades: 7, 8, 11, 12 (course specific)

Description: Organization of data of to examine measures of spread and measures of central tendency in examination of Gross Domestic Product (GDP) in Purchasing Power Standard (PPS) per inhabitant (2001).

Time: 45-50 minutes

Objectives: Building skills in exploratory data analysis: organize data for display in two formats (bar graph and box-and-whisker plot), determine measures of central tendency (mean, median, mode), measures of distribution (range, inter-quartile range, 5-point data summary), and introduce the fundamental idea of probability based on a data set.

Materials needed: straight edge, graph paper (for grades 7-8), paper for work, two different colored markers or highlighters, calculator (graphing calculator extension for high school students at the end of this lesson plan).

How Wealthy are Europeans?

Europeans have gained in wealth in the past decade and standards of living the EU's citizens have risen in all countries.¹ European Union funding has been instrumental in this achievement and similar growth is expected for those countries that joined the EU in 2004. However standards of living do vary from one region to another. A major purpose of the EU's structural funds is to assist poorer regions with economic growth in order to even out these differences.

In the table below are the values for the 2001 Gross Domestic Product (GDP) expressed in a common artificial currency called 'purchasing power standard' (PPS).¹

Wealth of Europeans by Member Country, 2001

Member Country	GDP in PPS per inhabitant	Member Country	GDP in PPS per inhabitant
Belgium	25,000	Luxembourg	45,400
Denmark	26,900	Netherlands	26,500
Germany	23,500	Austria	26,100
Greece	15,700	Portugal	16,500
Spain	19,700	Finland	24,300
France	24,500	Sweden	24,800
Ireland	27,500	United Kingdom	24,600
Italy	23,400	European Union	23,400

1. Arrange the countries and their respective GDP in numerical order from least to greatest in a column format on paper. You should have 15 countries listed.
2. Identify the median GDP of the EU member countries.
3. The mean GDP of the EU is given in the table. What is its value? How does the median value compare to the mean value of the GDP? Write 2-3 sentences to compare these values in your own words.
4. Draw a box around the median value in your list and label it **M**. There are 7 values less than the median and 7 remaining values greater than the median. Look at the 7 values greater than the median and find the median of these 7 values. Circle this value and call it **Q₃**. Look at the 7 values less than the median and find the median of those 7 values. Circle this value and call it **Q₁**.

5. Identify the smallest and largest values in your list. Put a star beside them. Label the smallest value at **MIN** and the largest value as **MAX**.
6. On a sheet of graph paper, draw a line to serve as the horizontal axis and label it from 0 to 50,000 in even increments of 2,500.
7. Pick one line on your graph paper just a few spaces above the axis line you just drew and plot the value for the MIN. Then plot the values for Q_1 , M, Q_3 and MAX. Draw a straight line from the MIN dot to the Q_1 dot and again from the Q_3 dot to the MAX dot. Draw a short vertical line through the dots for Q_1 , M and Q_3 . Connect these lines with horizontal lines at the top and bottom so that you create a box.

You have created a box-and-whisker plot diagram! The values for Q_1 , M and Q_3 represented the 3 quartile values. A quartile represents 25% of the values of a data set. The values less than Q_1 represent the lowest quartile. The values greater than Q_3 represent the highest quartile, or what is called the upper 25%.

8. a) What percent of the values are represented between the two quartile values?
b) Where is Q_2 ? How do we represent Q_2 ?
c) What does M represent in terms of probability?
d) What percent of countries are less than Q_1 ?
e) What percent of countries are greater than Q_3 ?
9. In a different color marker or highlighter, mark the mean GDP of \$23,400 for the EU on your box-and-whisker plot.
10. What do you notice about the values for Q_1 and Q_3 ? Where does the mean value for the EU fall on the graph?

11. The GDP for the United States for 2001 (the same year) was \$34,692.⁽²⁾ Can you compare this value to the mean GDP for the EU? Explain your answer.
12. The currency conversion rate from the Euro (the EU's monetary unit) to the US dollar was 0.8822 at the end of 2001.⁽³⁾ In other words, each American dollar is worth 0.8822 EU.
- How would you convert the \$34,692 into EUs?
 - How much is \$34,692 worth in EUs?
 - Plot your converted value in a different colored marker or highlighter on your box-and-whisker plot.
 - How does the United States GDP in 2001 compare to the EU GDP in 2001? Explain what you observe in a few sentences.

Extension for Further Exploration

Let's return to your box-and-whisker plot.

13. Are the whiskers the same length?

If they are not the same length, can you identify the value that makes one whisker longer than the other one?

Sometimes a very large or very small value can skew the data. We want to identify which possible values might be either very large or very small. These values are called outliers. We will first identify them visually, and then we will learn a mathematical formula for identifying these outliers. Sometimes it is easy to identify the values visually, and sometimes it is not, which is why there is a mathematical formula to help us.

On the other side of your graph paper, create a bar chart of the GDPs of each country. Make sure to label your horizontal and vertical axes. You may want to match the horizontal scale of your first graph (the box-and-whisker plot) where you start at 0 and increase each step on the graph by 2,500 up to the value of 50,000.

14. Can you identify at least one possible outlier from this graph? Which country does it represent?

15. Outliers are determined mathematically by a formula that uses some of the numbers on the box-and-whisker plot, namely Q_1 and Q_3 . The difference between Q_3 and Q_1 is called the inter-quartile range, or IQR.

a) What is the IQR for this data set?

An outlier at the upper end of the data set is found by the formula $Q_3 + (1.5 \times IQR)$. This is known as an upper bound. Any value larger than this computed value is an outlier!

b) Find the upper bound of your data by calculating $Q_3 + (1.5 \times IQR)$.

An outlier at the lower end of the data set is found by calculating $Q_1 - (1.5 \times IQR)$. This is known as a lower bound.

c) Find the lower bound of your data by calculating $Q_1 - (1.5 \times IQR)$.

d) What value(s) fall below the lower bound? What country or countries are represented by these values?

e) What value(s) fall above the upper bound? What country is represented here?

16. Outliers are normally not connected in the whiskers of the box-and-whisker plots. Redraw your box-and-whisker plot by leaving just dots to represent your outliers and shortening the whiskers to the next value above or below the outliers.

High School Exploration: Using the Graphing Calculator to Analyze Data & Build Boxplots

(Before starting, make sure there is nothing in the Y= editor.)

- 1) Enter data into a list: Stat→Edit→ enter data (2nd→ Mode when done)
- 2) Find the 5-number summary: Stat→Calc→<enter>1→2nd →1→enter.
Scroll down to find the MIN, Q₂, MEDIAN, Q₃, MAX
- 3) Graph the data: 2nd→Y= →<enter>→<enter ON>→tab over to the 4th graph under TYPE <enter>→Xlist: 2nd→1 (to enter list L₁ if not already on L₁). Freq = 1. Select one of the first two marks. (Third one does not show up well.) ZOOM→9 will give the graph.
- 4) TRACE will give the values at all key points: outliers (if present), ends of whiskers, Q₁, Median, Q₃. (The graph will be drawn to scale.)

Assessment:

Find the 5-number summary, draw the box-and-whisker plot and identify the outliers, both numerically and graphically.

Number of personal computers per 100 people in 2001 in EU member countries

Member Country	# personal computers per capita	Member Country	# personal computers per capita
Belgium	36	Luxembourg	45
Denmark	45	Netherlands	43
Germany	35	Austria	30
Greece	9	Portugal	22
Spain	17	Finland	43
France	34	Sweden	56
Ireland	39	United Kingdom	37
Italy	20	European Union	32

United States: 62 personal computers per 100 people

Japan: 35 personal computers per 100 people.

Sources:

(1) Key Facts and Figures about the European Union, page 23, European Commission, 2004.

(2) <http://eh.net/hmit/gdp/>

(3) <http://www.federalreserve.gov/releases/h10/20011231>

(4) Key Facts and Figures about the European Union, page 25, European Commission, 2004.

Standard Course of Study

Grade 7: Goal 4: Objectives 4.01, 4.02, 4.03 and 4.04

Grade 8: Goal 4: Objectives: 4.01, 4.03

Advanced Functions & Modeling (HS): Goal 1: 1.02a, 1.02c, 1.02e

Discrete Math: Goal 2: 2.01a, 2.01c, 2.01e

Integrated Math 3: Goal 3: 3.01a, 3.01c, 3.01e

AP Statistics: Goal 1: Objective 1.01; Goal 2: Objective 2.01; Goal 3: Objective 3.01